QUESTION BANK

GE – 1, CALCULUS

Q 1 The velocity of a particle moving in space is

$$\frac{d\mathbf{r}}{dt} = (\cos t)\,\mathbf{i} - (\sin t)\,\mathbf{j} + \mathbf{k}.$$

Find the particle's position as a function of t if $\mathbf{r} = 2\,\mathbf{i} + \mathbf{k}$ when $t = 0$.

Q 2 Find the length of one turn of the helix

$$\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}.$$

Q 3 Find the unit tangent vector of the helix

$$\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}.$$

Q 4 Find the unit tangent vector of the curve

$$\mathbf{r}(t) = (\cos t + t\sin t)\,\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j}, \qquad t > 0.$$

Q 5 Find $\mathbf{T}$ and $\mathbf{N}$ for the circular motion

$$\mathbf{r}\,(t) = (\cos 2t)\,\mathbf{i} + (\sin 2t)\,\mathbf{j}.$$

Q 6 Find the curvature for the helix

$$\mathbf{r}\,(t) = (a\cos t)\,\mathbf{i} + (a\sin t)\,\mathbf{j} + bt\,\mathbf{k}, \qquad a, b \geq 0,\ a^2 + b^2 \neq 0.$$

Q 7 Without finding $\mathbf{T}$ and $\mathbf{N}$, write the acceleration of the motion

$$\mathbf{r}(t) = (\cos t + t\sin t)\,\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j}, \qquad t > 0$$

Q 8

Show that the function

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}$$

has no limit as (x, y) approaches $(0, 0)$.

Q 9

Show that

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at every point except the origin

Q 10

Find f_x if $f(x, y) = \dfrac{2y}{y + \cos x}$.

Q 11

Find $\partial z / \partial x$ if the equation

$$yz - \ln z = x + y$$

defines z as a function of the two independent variables x and y and the partial derivative exists.

Q 12

Find $\partial^2 w / \partial x \, \partial y$ if

$$w = xy + \frac{e^y}{y^2 + 1}.$$

Q 13

Express $\partial w / \partial r$ and $\partial w / \partial s$ in terms of r and s if

$$w = x + 2y + z^2, \qquad x = \frac{r}{s}, \qquad y = r^2 + \ln s, \qquad z = 2r.$$

Q 14

Use the Chain Rule to find the derivative of

$$w = xy$$

with respect to t along the path $x = \cos t$, $y = \sin t$. What is the derivative's value at $t = \pi/2$?

Q 15 Find the derivative of

$$f(x, y) = x^2 + xy$$

at $P_0(1, 2)$ in the direction of the unit vector $\mathbf{u} = \left(1/\sqrt{2}\right)\mathbf{i} + \left(1/\sqrt{2}\right)\mathbf{j}$.

Q 16 Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j}$.

Q 17 Find the directions in which $f(x, y) = (x^2/2) + (y^2/2)$ (a) increases most rapidly and (b) decreases most rapidly at the point $(1, 1)$. (c) What are the directions of zero change in f at $(1, 1)$?

Q 18 Find an equation for the tangent to the ellipse

$$\frac{x^2}{4} + y^2 = 2$$

) at the point $(-2, 1)$.

Q 19

a) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

b) In what directions does f change most rapidly at P_0, and what are the rates of change in these directions?

Q 20 Find the tangent plane and normal line of the surface

$$f(x, y, z) = x^2 + y^2 + z - 9 = 0 \qquad \text{A circular paraboloid}$$

at the point $P_0(1, 2, 4)$.

Q 21 The surfaces

$$f(x, y, z) = x^2 + y^2 - 2 = 0 \qquad \text{A cylinder}$$

and

$$g(x, y, z) = x + z - 4 = 0 \qquad \text{A plane}$$

meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0(1, 1, 3)$.

Q 22 Find the plane tangent to the surface $z = x \cos y - ye^x$ at $(0, 0, 0)$.

Q 23 Estimate how much the value of

$$f(x, y, z) = xe^y + yz$$

will change if the point $P(x, y, z)$ moves 0.1 unit from $P_0(2, 0, 0)$ straight toward $P_1(4, 1, -2)$.

Q 24 Find the local extreme values of the function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4.$$

Q 25 Find the local extreme values of $f(x, y) = xy$.

Q 26 Find the absolute maximum and minimum values of

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

on the triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.