Design of Experiments

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- To really understand cause-and-effect relationships in a system you must deliberately change the input variables to the system and observe the changes in the system output that these changes to the inputs produce. In other words, you need to conduct experiments on the system.
- We can define an ***experiment*** as a test or series of runs in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response.

- We may want to determine which input variables are responsible for the observed changes in the response, develop a model relating the response to the important input variables and to use this model for process or system improvement or other decision-making.
- Statistical design of experiments refers to the process of planning the experiment so that appropriate data will be collected and analyzed by statistical methods, resulting in valid and objective conclusions. The statistical approach to experimental design is necessary if we wish to draw meaningful conclusions from the data. When the problem involves data that are subject to experimental errors, statistical methods are the only objective approach to analysis.

- Creation of controlled conditions is the main characteristic feature of experimentation and DOE specifies the nature of control over the operations in experiments. Proper designing ensures that the assumptions required for appropriate interpretations of data are satisfied thus increasing the accuracy and sensitivity of results.
- There are two aspects to any experimental problem: the design of the experiment and the statistical analysis of the data. These two subjects are closely related because the method of analysis depends directly on the design employed.

Guidelines for Designing an Experiment

Pre-experimental Planning

- 1. Recognition of and statement of the problem
- 2. Formulation of hypothesis
- 3. Selection of the response variable
- 4. Choice of factors, levels, and ranges
- 5. Choice of experimental design
- 6. Performing the experiment
- 7. Statistical analysis of the data
- 8. Conclusions and recommendations
- 9. Evaluation of the whole investigation

In practice, steps 3 and 4 are often done simultaneously or in reverse order.

Experiments

There are three main types of experiments:

- Varietal trials
- Factorial experiments
- Bio-assays

Different levels of one factor usually form the treatments in varietal trials. The main purpose of the varietal trials is to compare the treatments in all possible pairs. Eg. Different doses of a drug under investigation, different varieties of a crop etc.

Factorial Experiments

- In factorial experiments combinations of two or more levels of more than one factors are treatments. One can study main effects and interaction effects as well in this case. For eg. One factor be Nitrogen with two levels n1 and n2 and another factor be irrigation with three levels i1, i2 and i3 then one can form 6 combinations i1n1, i2n1, i3n1, i1n2, i2n2, i3n2. such combinations form treatments in factorial experiments.

Three Principles of Design

- Prof. R.A. Fisher gave three basic principles of experimental design
- replication,
- randomization and
- Blocking or Local Control .

Replication

- By replication we mean an independent repeat run of each factor combination. Replication has two important properties. First, it allows the experimenter to obtain an estimate of the experimental error. This estimate of error becomes a basic unit of measure-ment for determining whether observed differences in the data are really statistically different. Second, if the sample mean (ybar) is used to estimate the true mean response for one of the factor levels in the experiment, replication permits the experimenter to obtain a more precise estimate of this parameter.

Replication

For example; if σ^2 is the variance of an individual observation and there are *n* replicates, the variance of the sample mean is

$$\sigma_{y}^{2} = \sigma^2/n$$

We see that precision of the estimate is inversely proportional to the number of replications. The more te number of replications higher the precision of the estimate.

- But increased number of replications may increase the variance. Typically a large number of experimental units are more variable than a small number, so increasing the replications may increase the value of σ^2, sometimes tis increase in σ^2 may overweigh the increase in no. of replications.
- Increased replications usually raise the power because it usually raises the no. of degrees of freedom of residuals.

- Increase in the number of replications also increases the cost of experimentation.
- Thus, factors that determine the number of replications:
- 1. Uniformity of experimental units
- 2. Experimental designs
- 3. Degree of precision required
- 4. Number of treatments
- 5. Time allotment
- 6. Cost and availability of funds or resources

Randomization

- One major problem that is encountered during experimentation is avoiding biased selection of experimental materials, which results in inaccurate or misleading experimental data. To avoid such bias, Fisher introduced the principle of randomization. This principle states that before an effect in an experiment can be ascribed to a given cause or treatment independently of other causes or treatments, the experiment must be repeated on a number of control units of the material and that all units of material used in the experiments must be randomly selected samples from the whole population they are intended to represent. Thus randomisation is use of chance to divide experimental units into groups
- It is a process of assigning the treatments among the experimental units such that every treatment has equal chance of being assigned to any experimental unit
- Randomizations purpose is to remove bias and other sources of extraneous variation, which are uncontrollable. It is the basis of any valid statistical test.

- Effective randomization gives rise to good representativeness in the samples and balance among the research subjects in each group for important non-experimental factors.
- Thus randomization ensures – validity of results; helps in having objective comparison among treatments; it ensures independence of observations which is necessary for drawing valid inferences by applying appropriate statistical techniques.

Local Control

- Blocking is a design technique used to improve the precision with which comparisons among the factors of interest are made. Often blocking is used to reduce or eliminate the variability transmitted from nuisance factors—that is, factors that may influence the experimental response but in which we are not directly interested.

- Local control, like replication is yet another device to reduce or control the variation due to extraneous factors and increase the precision of the experiment. If, for instance, an experimental field is heterogeneous with respect of soil fertility, then the field can be divided into smaller blocks such that plots within each block tend to be more homogeneous. This kind of homogeneity of plots (experiment units) ensures an unbiased comparison of treatment means, as otherwise it would be difficult to attribute the mean difference between two treatments solely to differences between treatments when the plot differences also persist. This type of local control to achieve homogeneity of experimental units, will not only increase the accuracy of the experiment, but also help in arriving at valid conclusions.

- In short Local control s dividing the experimental material into blocks which are homogeneous within and heterogeneous in between. The variation among these blocks is eliminated from the error and thereby efficiency is increased.
- Choice of number of replications reduces standard error of the estimates of the treatment effects because the standard error of the estimate of the difference between treatment effects is $\sqrt{\frac{2\hat{\sigma}^2}{n}}$ But its choice can not reduce the error var itsel: even though a large number of replications can ensure a more stable estimate.

- By using local control it is possible to reduce the error variance itself. It usually raises power as it decreases the degrees of freedom of residuals.

Experimental Error

- It is an estimate of the inherent uncertainty associated with our experimental procedure, and is not dependent on any presumed ‘right answer’.
- Total variation in an experiment can be divided into two parts explained systematic variation and unexplained random variation. This unexplained random variation is called experimental error.

- Besides the variations produced in the observations due to known sources, the variations are also produced by a large number of unknown sources such as uncontrolled variation in extraneous factors related to the environment, genetic variations in the experimental material other than that due to treatments, etc. They are there, unavoidable and inherent in the very process of experimentation. These variations because of their undesirable influences are called *experimental error* thereby meaning not an arithmetical error but variations produced by a set of unknown factors beyond the control of the experimenter. These may even be errors associated with measurement.

- Experimental error provides a basis for te confidence to be placed in the inference about the population. So it is important to estimate and control experimental error.

Terminology

- Before conducting an experiment, an ***experimental unit*** is to be defined. For example, a leaf, a tree or a collection of adjacent trees may be an experimental unit. An experimental unit is also sometimes referred as *plot.*
- ***Factors*** are variables that are controlled and varied during the course of the experiment. Eg. pressure and temperature are factors in a chemical experiment.
- Any substance or item whose effect on the data is to be studied is called ***Treatment Factor .***

- ***Response Variable or Outcome*** This is what we measure on each experimental unit. Eg.Blood pressure in humans ;Mortality rate for young turkeys; Yield of a crop; Average score on a final exam; Average hardness of cooki
- Specific types or amounts of the treatment factor that will actually be used in the experiment are called ***Treatment Levels***. A factor may comprise of 3 different amounts of fertilizer or may be 4 different amounts of irrigation or 2 harvesting regimes.
- A collection of plots is termed a ***block***. A group of experimental units which share a common characteristic.
- The characteristic used to create the blocks is called a ***Blocking Factor***
- ***Treatment Combinations*** – combination of the levels of different treatment factors.

- In many experiments, including clinical trials with people, new treatments are compared with a standard treatment or no treatment at all. ***Control*** is the term that is used to refer to that standard treatment or no treatment.

Uniformity Trials

- In a uniformity trial, a particular variety of crop is sown on the entire experimental field and uniformly managed throughout the growing season. At the time of harvest a substantial boarder is removed from all sides of the field. The remainder of the field is divided into small plots which are termed as basic units, with the same dimensions. The produce from these basic units is harvested and recorded separately for each basic unit. Then the mean yield per basic unit is computed.
- The usefulness of a uniformity trail lies in the fact that neighbouring units may be amalgamated to form larger plots of various sizes and shapes. The variation in yield over the field due to soil heterogeneity and other manual errors generally summed up in the term "Experimental Error" may be calculated for each type of plot thus formed. Hence all efforts in designing field experiments are directed to measure and control this source of variation. So it was thought proper to review the post literature with these aspects in mind. Here, an attempt is made to compile the contributions made by different authors about the different methodologies in arriving at optimum plot size and shape its subsequent utility in laying out of different statistical designs. The determination of optimum plot size is a vital factor in field experimentation as it is a fundamental step. Because of the variability it introduces both due to crop species and soil heterogeneity, it in turn engages the attention of agricultural research scientists. It is highly vulnerable to adophic, climatic and biotic factors.
- The soil not being uniform posses much problem in agricultural experimentation. The criterion of field heterogeneity is of the greatest value as it will be universally applicable, for comparing different species, characters and experiments. So having decided to cope up with the problem, attempts were first directed to measure the degree of variation and then to control it (Harris, 1920).

- Uniformity trials involves growing a particular crop on a field or piece of land with uniform conditions. All sources of variation except that due to native soil difference are kept constant. At the time of harvest the entire field is divided into smaller units of same size and shape and the produce from each such unit is recorded separately. The smaller the basic units, the more detailed is the measurement of soil heterogeneity.

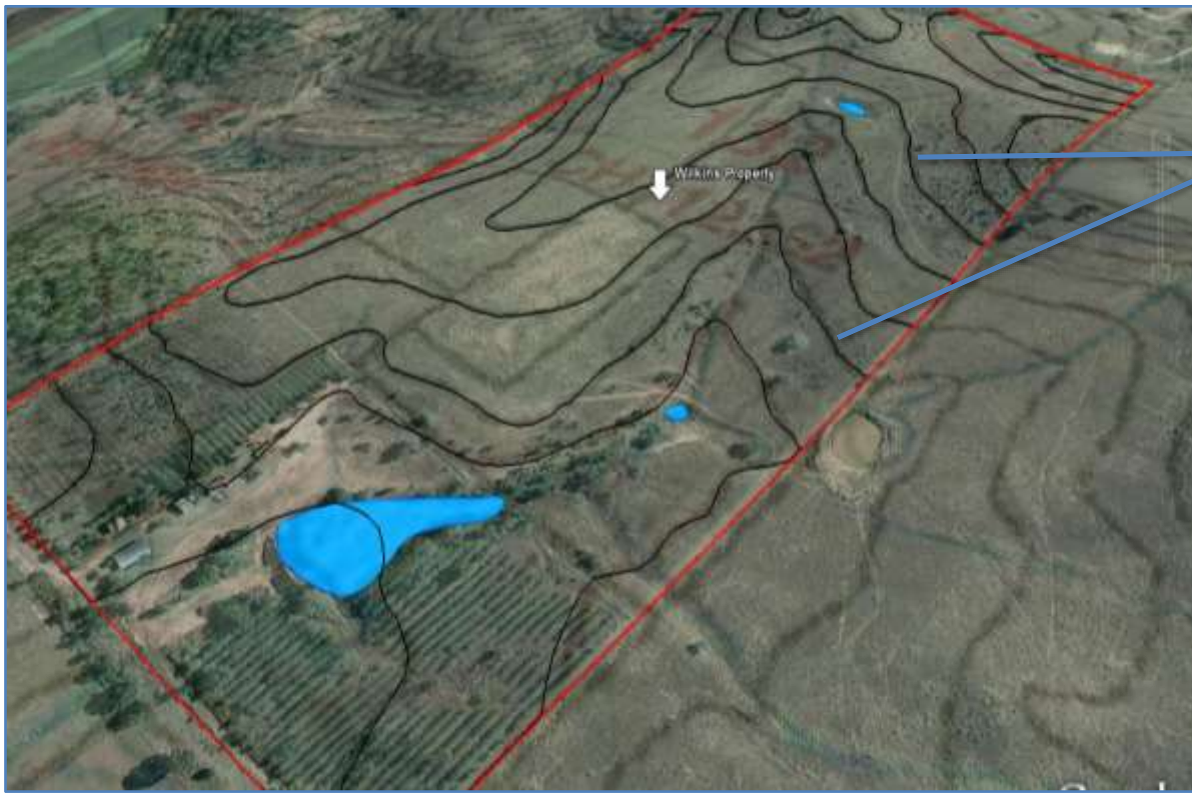
Uses of Uniformity Trials

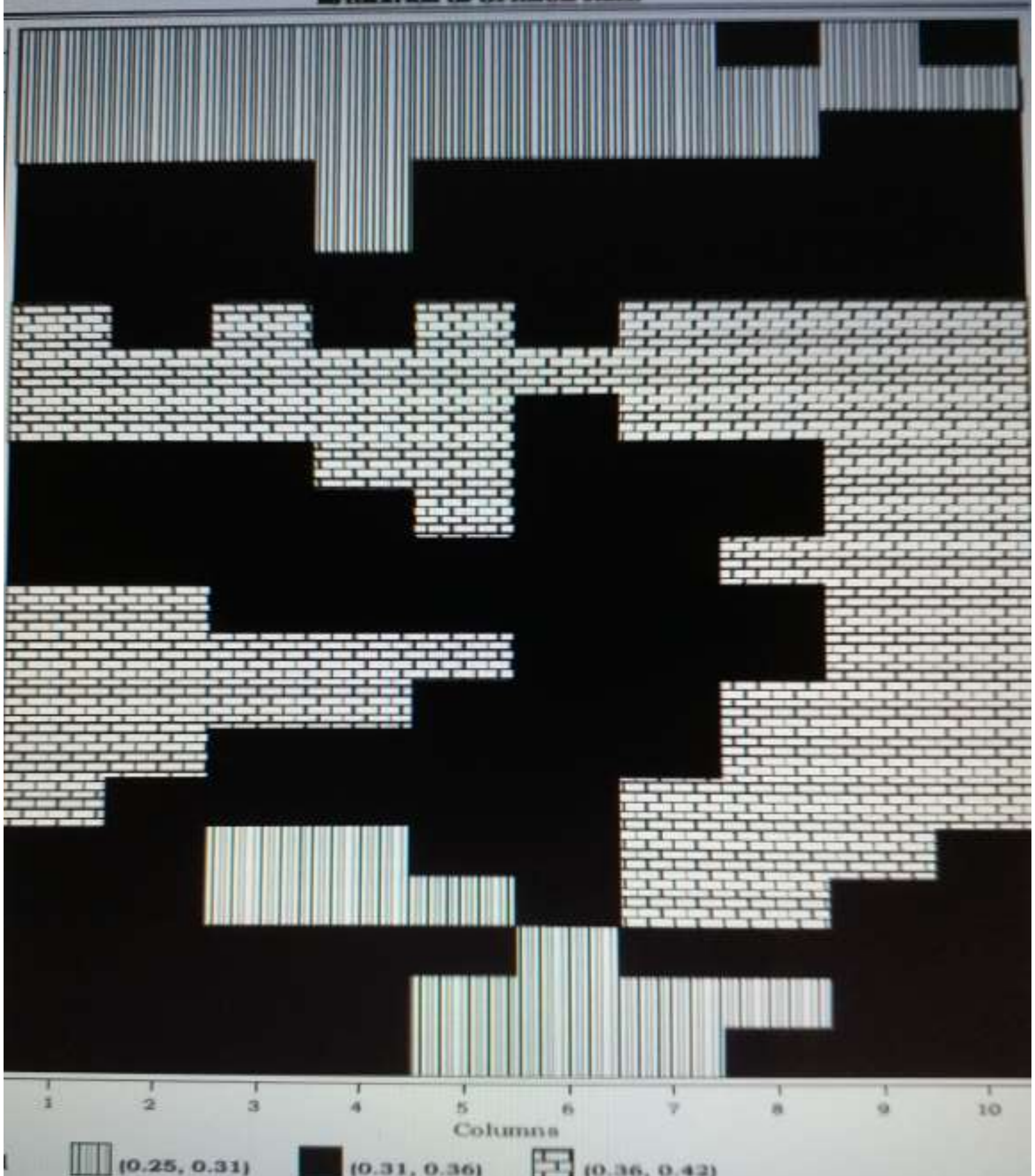
- Gives us an idea about fertility gradient
- Determines the Shape of the plots to be used
- Determines optimum size of plots
- Estimation of the number of replications required for achieving certain degree of accuracy.
- Give a graphic picture of the variation of sol fertility.

Fertility Contour Maps

- An approach to describe the heterogeneity of land is to construct the fertility contour map. This is constructed by taking the moving averages of yields of unit plots and demarcating the regions of same fertility by considering those areas, which have yield of same magnitude. Accordingly the field can be divided into relatively homogeneous subgroups to control experimental error.

Contour Map





1. Fertility contour map of a field measuring 24m×12m basic units

- The values of the vertical and horizontal serial correlation coefficients are 0.314 and 0.341 respectively. Both coefficients are low which indicates that some fertility gradient is present. However, the horizontal serial correlation coefficient was little high than vertical implying that the fertility gradient was more pronounced horizontally than vertically.

Fairfield Smith's Variance Law

Smith in 1938 gave an empirical relationship between variance and plot size. He developed an empirical model representing the relationship between plot size and variance of mean per plot.

$$V_x = V_1/x^b \text{ or } \quad \log V_x = \log V_1 - b \log x$$

where x is the number of basic units in a plot, V_x is the variance of mean per plot of x units, V_1 is the variance of mean per plot of one unit and b is the characteristics of soil and measures correlation among contiguous units

- If b = 1; $V_x = V_1 / x$ and the units making up the plots of x unit are not correlated at all.
- On the other hand if b = 0 the x units are perfectly correlated and $V_x = V_1$ (constant) so there is no gain due to larger size of plot.
- In general b will lie between 0 and 1 so that the larger plot gives more information with the same number of plots. In that case larger area for the purpose of experimentation will be used. The values of V_1 and b are determined by principle of least squares.

Suppose the total experimental area is *rx* units and we use plots of sizes as follows:

Size of plot	X	x/2	…	x/k
No of replicates	*r*	*2r*	…	*kr*
Variance of mean yield of a treatment	V_x/r	$V_{x/2}/2r$	…	$V_{x/k}/kr$

Therefore, variance of the mean yield decreases with increase in value of *k*. This suggest that one should use plots of smaller sizes for increase in precision of experiments.

Size and Shape of plots

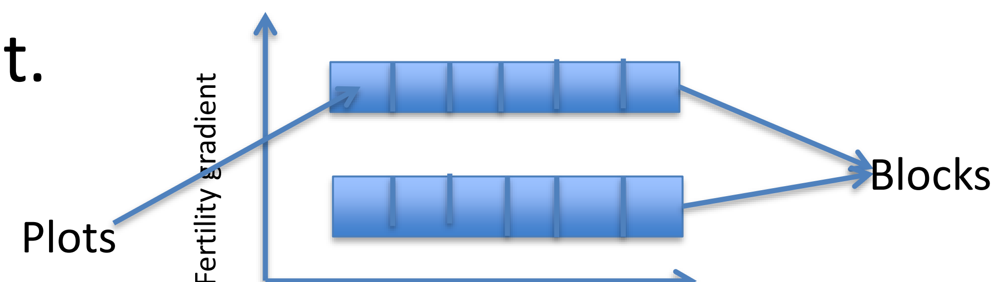
- The size and shape of the plots are determined on the basis of the experimental area provided, the type of material to be tested and type of precision required etc.
- If the plot size is small the shape may have little or no effect on experimental precision, whereas for larger plot sizes the effect of shape may be considerable.
- The precision of significance tests in field trial is largely controlled by size and shape of plots, which are further controlled by the size, and shape of area available for the particular trial, the nature of fertility or other variation.

Size of Plots

- Experimental material available determine plot size. Eg. In animal experiments the pen’s or cages are already constructed and not easily changed, pastures ad paddock size may be determined by those already available and fenced, ig green house studies the experimental units may have to be small etc.
- Nature of Experimental material- plot size required for oat may be different from corn, pen or cage size for chicken or cattle etc.
- Number of treatments per block.

Shape of plots

- Long and narrow plots have been found to be relatively more precise because neighbouring plots are then very close together.
- If the fertility gradient is known, then arrangement of experimental units is to be made by forming blocks of homogeneous plots for Maximum precision by arranging the plots in a block with their long side parallel to the direction of fertility gradient and blocks one after the other in the direction of the gradient.



- On particular types of land, other shapes may also be feasible, e.g. plots following the shape of the land contour in terraced land.
- However, this all assumes that there is no need for border areas round each plot, because of the possibility of some carry-over of treatment effects from neighbouring plots. Where guard rows are needed, both small plots and narrow plots become inefficient, because they result in a greater proportion of the plot being wasted.
- In the absence of all criteria it is better to have square shaped plots and they should be of such a size that most of the plot can be harvested and recorded.