Unique Paper Code : **62351101_OC**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.) I Year**

Semester : **I**

Duration : **3 Hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find the limit of the given functions

(i) $\lim_{x\to 0}\frac{x\cos x-\sin}{x^2\sin x}$ (ii) $\lim_{x\to 0}\frac{1}{1+e^{1/x}}$.

Discuss the continuity and discontinuity of following functions,

(i) $f(x)=|x-1|+|x-2|$ at $x=1$ & $x=2$

(ii) $f(x)=\begin{cases}\frac{xe^{1/x}}{1+e^{1/x}}, & \text{if } x\neq 0,\\ 0, & \text{if } x=0,\end{cases}$ at $x=0$.

Can a function have more than one limit? Explain.

2. If $y=\sin mx+\cos mx$, then show that $y_n=m^n[1+(-1)^n sin\,2mx]^{1/2}$.

State Leibnitz's theorem. If $y=\sin^{-1}x$ then prove that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0.$$

If $u=x^2\tan^{-1}\left(\frac{x}{y}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$ then prove that

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}.$$

3. Prove that the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2021$ touches the straight line $\frac{x}{a}+\frac{y}{b}=2$ at the point (a,b), whatever be the value of n.

Show that normal at any point of the curve

$$x=a\cos\theta+a\,\theta\sin\theta,\ y=a\sin\theta-a\,\theta\cos\theta$$

is at constant distance from origin.

Find the curvature at any point θ on the curve

$$y=a\left(\cos\theta+\log\left(\tan\left(\frac{\theta}{2}\right)\right)\right), y=a\sin\theta.$$

4. Find asymptotes of the curve $(y-3)^2(x^2-9) = x^4 + 81$.

Determine the position and nature of double points on the curve

$$x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0$$

Trace the curve $x^2y^2 = x^2 - 25a^2$.

5. State Lagrange's mean value theorem and give its geometrical interpretation. Applying Lagrange's mean value to the function defined by $f(x) = \log(1+x)\ \forall\ x > 0$ show that

$$0 < [\log(1+x)]^{-1} - x^{-1} < 1 \text{ whenever } x > 0.$$

Separate the interval in which the function defined on $\mathbb{R}$

$$f(x) = 2x^3 - 15x^2 + 36x - 1$$

is increasing or decreasing.

Prove that $\tan x > x$ whenever $0 < x < \pi/2$.

6. Obtain Maclaurin's series expansion of $\cos 2x$.

Evaluate $\lim_{x\to 0} \frac{sinx - x}{x\, sinx}$

Find the maximum value of $\left(\frac{1}{x}\right)^x$.