This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 7947

Unique Paper Code : 62351101 HC

Name of the Paper : Calculus

Name of the Course : B.A. (Prog.) Mathematics

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Discuss the existence of the limit of the function : 6

$$f(x)=\frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$$

at $x = 0$.

P.T.O.

(*b*) Find the value of k for the function :

$$f(x) = \begin{cases} 2x - 1 ; & x < 0 \\ kx + 2 ; & x \geq 0 \end{cases}$$

to be continuous at $x = 0$.

(*c*) Examine the following function for differentiability at $x = 1$ and $x = 2$:

$$f(x) = |x - 1| + |x - 2|.$$

2\. (*a*) Find the *n*th derivative of $(\sin x)^4$.

(*b*) If $y = (\sin^{-1} x)^2$, prove that :

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0.$$

(*c*) If

$$z = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right),$$

then using Euler's theorem, prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z.$$

3\. (*a*) Find the equation of tangent and normal at the given point to the curve :

$$x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta)$$

at $\theta = \pi/2$. 6½

(*b*) Find the length of the perpendicular from the foot of the ordinate on any tangent to the curve : 6½

$$y = c\cosh\left(\frac{x}{c}\right).$$

(*c*) Find the radius of curvature at the point $(3a/2, 3a/2)$ on the curve : 6½

$$x^3 + y^3 = 3axy.$$

4\. (*a*) Find the asymptotes of the curve : 6½

$$(x + y)(x - y)(x - 2y) + 4y(2x - y) + 4y = 0.$$

(*b*) Show that the origin is a node, cusp, or conjugate point on the curve : 6½

$$(x^2 + y^2 - bx)^2 - a^2(x^2 + y^2) = 0$$

according as $a < b$, $a = b$, $a > b$.

(c) Trace the curve :

$$y^2x = a^2(a - x).$$

5. (a) State Lagrange's mean value theorem. Using it, prove that if a function f satisfies the hypothesis of Lagrange's mean value theorem on $[a, b]$, and if $f'(x) = 0$ for all $x \in]a, b[$, then there exist a constant $k \in \mathbf{R}$ such that $f(x) = k$ for all $x \in [a, b]$.

(b) State Taylor's theorem with Cauchy's form of remainder. Obtain the Cauchy's remainder after $(n + 1)$th term of the function $f(x) = \cos x$ of all $x \in [-\pi, \pi]$.

(c) Let $f(x) = \log(1 + x)$ for all $x \geq 0$. Using Lagrange's mean value theorem for the function f, show that :

$$0 < \log(1 + x)^{-1} - x^{-1} < 1, \ \forall \ x > 0.$$

6. (a) Define an extremum of a function. Give an example of a function f which has extreme values without being differentiable at a point. Justify your answer. 6½

(b) Find the values of a and b such that : 6½

$$\lim_{x\to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3} = 1.$$

(c) Let f be a function defined by : 6½

$$f(x) = \frac{ax + b}{cx + d} \quad \forall \ x \in \mathbf{R}\backslash\{-d/c\} \text{ and } ad - bc \neq 0.$$

Does the function f have any extremum. Justify your answer.