Semester 5

DEPARTMENT OF STATISTICS

B. Sc. (H) Statistics

Category I

DISCIPLINE SPECIFIC CORE COURSE – 13: THEORY OF ESTIMATION

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Theory of Estimation	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of probability, probability distributions and sampling distributions

Learning Objectives

The learning objectives include:

- Characterisation of the population based on sample information
- Understanding process of learning and determining the population characteristics based the available data.
- Strength and weakness of various methods for obtaining point and interval estimators with respect to optimal/desirable properties.

Learning Outcomes:

After completing this course, students will develop a clear understanding of:

- List desirable properties of point estimators based on an unknown parameter of a distribution viz. Unbiasedness, Consistency, Efficiency and Sufficiency.
- Derive the UMVUE of a parameter or function of a parameter (Using Cramer- Rao inequality, Rao-Blackwell theorem, and Lehmann- Scheffé Theorem).
- Understand and apply different techniques of finding optimal point estimators such as Maximum Likelihood Estimation, Method of Least Squares, Method of moments and the method of minimum chi-Squares

- Construct interval estimators, pivot method (Confidence Intervals) for unknown population parameters.

SYLLABUS OF DSC-13

Theory

UNIT I **(18 hours)**

Estimation

Estimation: Concepts of estimation, unbiasedness, sufficiency, consistency and efficiency. Fisher-Neyman Criterion (statement and applications), Factorization theorem. Complete statistic, Minimum variance unbiased estimator (MVUE), Rao-Blackwell and Lehmann-Scheffe theorems and their applications. Cramer-Rao inequality, Minimum Variance Bound estimators (MVBE) and their applications.

UNIT II **(10 hours)**

Methods of Estimations

Methods of Estimation: Method of moments, method of maximum likelihood estimation and method of minimum Chi-square.

UNIT III **(12 hours)**

Interval estimation

Interval estimation - Confidence intervals for parameters of various distributions, confidence interval for Binomial proportion, confidence interval for population correlation coefficient for Bivariate Normal distribution, pivotal quantity method of constructing confidence intervals, shortest length confidence intervals, large sample confidence intervals.

UNIT IV **(5 hours)**

Censored Data

Failure censored samples, time censored sample, estimation of expected lifetime in failure censored samples for one parameter exponential lifetime distribution

PRACTICAL/LABWORK (30 hours):

List of Practical

1. Unbiased estimators (including unbiased but absurd estimators)
2. Consistent estimators, efficient estimators and relative efficiency of estimators.
3. Cramer-Rao inequality and MVB estimators
4. Sufficient Estimators – Factorization Theorem, Complete Sufficient estimators, Rao-Blackwell theorem.
5. Lehman-Scheffe theorem and UMVUE
6. Maximum Likelihood Estimation
7. Asymptotic distribution of maximum likelihood estimators
8. Estimation by the method of moments,
9. Estimation by method of minimum Chi-square
10. Confidence interval based on large sample test
11. Confidence interval based on exact sample test

Practical work to be conducted using electronic spreadsheet / EXCEL/ Statistical Software Package/ SPSS/ calculators.

ESSENTIAL READINGS:

- Goon, A.M.; Gupta, M.K.; Dasgupta, B. (2013).: An Out Line of Statistical Theory, Volume 2, The World Press, Kolkata.
- Gupta, S.C. and Kapoor, V.K.(2020): Fundamental of Mathematical Statistics, 12th Edn. Sultan Chand and Sons.
- Sinha, S.K. (1986):Reliability and Life testing; Wiley Eastern.

SUGGESTIVE READINGS:

- Hogg, R.V. and Craig, A.T (2018): Introduction to Mathematical Statistics, 8th Edn. Pearson Education.
- Casella, G. and Berger, R.L. (2002): Statistical Inference. 2nd Edition, Duxbury Press, Pacific Grove.
- Hogg, R.V. and Tanis, E.A. (1988): Probability and statistical Inference, 6th Edn. Pearson Education
- Rohatgi V.K, (2013): Statistical Inference- Dover Publication, New York.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE-14: LINEAR MODELS

CREDIT DISTRIBUTION, ELIGIBILITY, AND PRE-REQUISITES OF THE COURSE

Course title & code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lectures	tutorials	practical		
Linear Models	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of matrix theory, probability distributions and sampling distributions

Learning Objectives:

learning objectives include:

- Developing a clear understanding of the fundamental concepts of linear models.
- Developing associated skills allowing the students to work effectively with them.

Learning Outcomes:

After completion of this course, students will develop a clear understanding of:

- Theory and estimation of Linear Models.
- Gauss-Markov Theorem and its use.
- Distribution of quadratic forms.
- Simple and Multiple linear regression models and their applications.

- Fitting of these models to real or synthetic data, derivation of confidence and prediction intervals, and a sound scientific interpretation of the results.
- Techniques of Analysis of Variance and Covariance under fixed effects model.
- Assessment of the quality of the fit using classical diagnostics, awareness of potential problems (outliers, etc.) and application of remedies to deal with them.

SYLLABUS OF DSC-14

THEORY

UNIT I **(10 Hours)**

Estimation theory and Distribution of Quadratic forms

Gauss-Markov setup, Theory of linear estimation, Estimability of linear parametric functions, Method of least squares, Gauss-Markov theorem, Estimation of error variance. Cochran's theorem and distribution of quadratic forms.

UNIT II **(10 Hours)**

Analysis of Variance

Definition of fixed, random, and mixed effect models, Technique of ANOVA, assumptions for its validity, analysis of variance in one-way classified data and in two-way classified data with an equal number of observations per cell for fixed effect models.

UNIT III **(14 Hours)**

Regression analysis:

Estimation and hypothesis testing in case of simple and multiple linear regression analysis, Confidence intervals, and Prediction intervals, Concept of model matrix and its use in estimation. Effect of orthogonal columns in the X matrix, Partial F-test and Sequential F-test, Bias in regression estimates.

UNIT IV **(4 Hours)**

Analysis of Covariance:

Technique of ANOCOVA, assumptions for its validity, use, and analysis of covariance in one-way classified data with a single concomitant variable.

UNIT V **(7 Hours)**

Model checking and Model Building

Prediction from a fitted model, Residuals and Outliers, Lack of fit and pure error, Violation of usual assumptions concerning normality, Homoscedasticity, and collinearity, Diagnostics using quantile-quantile plots. Techniques for Variable selection. Polynomial Regression models: Orthogonal Polynomials.

PRACTICAL/LABWORK -30 Hours

List of Practicals

1. Estimability when X is a full rank matrix.
2. Estimability when X is not a full rank matrix.
3. Distribution of Quadratic forms.
4. Simple Linear Regression.
5. Multiple Regression.
6. Tests for Linear Hypothesis.
7. Bias in regression estimates.
8. Lack of fit.
9. Stepwise regression procedure.

10. Analysis of Variance of a one-way classified data.
11. Analysis of Variance of two-way classified data with one observation per cell.
12. Analysis of Variance of two-way classified data with m (> 1) observations per cell.
13. Analysis of Covariance of a one-way classified data.
14. Residual Analysis.
15. Orthogonal Polynomials.

Practical work to be conducted using electronic spreadsheet / EXCEL/ Statistical Software Package/ SPSS/ calculators.

ESSENTIAL READINGS

- Montgomery, D. C., Peck, E. A. and Vining, G. G. (2012): Introduction to Linear Regression Analysis, 5th Ed., John Wiley and Sons.
- Rencher, A. C. and Schaalje, G. B. (2008): Linear Models in Statistics, 2nd Ed., John Wiley and Sons.
- Draper, N. R. and Smith, H. (1998): Applied Regression Analysis, 3rd Ed., John Wiley and Sons.

SUGGESTIVE READINGS:

- Weisberg, S. (2005): Applied Linear Regression, 3rd Ed., John Wiley and Sons.
- Rawlings, John O. Pantula Sastry G. Dickey, David A. (1998) Applied Regression Analysis: A Research Tool, Second Edition
- Bapat, R.B.(1993): Linear Algebra and Linear Models, Hindustan Book Agency.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch University of Delhi, from time to time.

DISCIPLINE SPECIFIC CORE COURSE 15 –: STOCHASTIC PROCESSES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Stochastic Processes	4	3	0	1	Class XII pass with Mathematics.	Knowledge of probability, probability distributions, and sampling distributions

Learning Objectives:

- To define, design and model

- To analyze transitions through Markov chains
- To identify the real life applications of stochastic processes

Learning Outcomes:

After completing this course, students will develop a clear understanding of:

- The fundamental concepts of stochastic processes.
- Tools needed to analyze stochastic processes.
- Markov processes and Markov chains.
- Markov chain applications.
- Poisson process and its variations.
- Random walk and ruin theory

SYLLABUS OF DSC-15

Theory

UNIT I **(13 hours)**

Introduction of Stochastic Process

Probability Distributions: Generating functions, Bivariate probability generating functions, and their application.

Stochastic Process: Introduction, Covariance stationary, and Stationary Process.

UNIT II **(15 hours)**

Markov Chains

Markov Chains: Definition of Markov Chain, transition probability matrix, order of Markov chain, Markov chain as graphs, higher transition probabilities.

Classification of states and chains, stationary process, and stability of Markov system. Generalization of independent Bernoulli trials,

UNIT III **(12 hours)**

Poisson Process

Poisson Process: postulates of Poisson process, and properties of Poisson process and applications.

Gambler's Ruin Problem: Classical ruin problem, expected duration of the game.

PRACTICAL/LAB WORK – (30 hours)

List of Practical:

1. Applications of Partial Fraction Theorem.
2. Problems based on (covariance) stationary processes.
3. Simulation of Markov chains.
4. Calculation of transition probability matrices.
5. To check whether the given chain is irreducible or not.
6. Classification of states.
7. Computation of probabilities in case of generalizations of independent Bernoulli trials.
8. Simulation and applications of Poisson processes.
9. Transition Markov chain in case of gambler's ruin problem .
10. Calculation of probabilities for ruin problems.

Practical work to be conducted using electronic spreadsheet / EXCEL/ Statistical Software Package/ SPSS/ calculators.

ESSENTIAL READINGS

- Feller, W. (1968). Introduction to probability Theory and Its Applications, Vol I, 3rd Ed., Wiley International.
- Medhi, J. (2019). Stochastic Processes,4th Ed., Reprint, New Age International Publishers.

SUGGESTIVE READINGS:

- Sheldon M. Ross (2007) : Introduction to Probability Models, 9th edition, Academic Press publications
- Karlin & Taylor (1975) : A first course in stochastic processes, 2nd edition, Academic Press publications
- Basu, A.K. (2005). Introduction to Stochastic Processes, Narosa Publishing.
- P. G. Hoel, S. C. Port and C. J. Stone: Introduction to Stochastic Processes.
- J. G. Kemeny, J. L. Snell and A. W. Knapp: Finite Markov Chains.
- Geoffrey R, Grimmett & David R. Stirzaker : Probability and Random Processes
- Bhat,B.R. (2000). Stochastic Models: Analysis and Applications, New Age International Publishers.

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