Unique paper Code	:	32371303
Name of the Paper	:	Mathematical Analysis
Name of the Course	:	B.Sc. (H) Statistics Under CBCS
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	75

Instructions for Candidates

1. **Attempt any four questions.**
2. **All Questions carries equal marks ~~18.75 marks each~~.**

1. (a) Define

(i) Neighbourhood of a point,
(ii) Open Set and
(iii) Closed Set.

Give an example of each of above.

Also show that the union of an arbitrary family of Closed sets may

(i) be a Closed Set
(j) fail to be a Closed Set

(b) Define Supremum and infimum of a Set. Find ~~these~~ supremum and infimum for the following set ~~S where~~

$$S = \{ x \in R: x^2 - 2x - 5 < 0 \}.$$

(c) Examine the following set for its limit point/~~(~~s~~)~~

$$S = \left\{ \frac{1}{n} , \; n \in Z^+ \right\}.$$

~~(6, 4, 8.75)~~

2. (a) Define a Convergent Sequence. Use the definition to show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

(b) Define Convergence and Absolute Convergence of a Series. Is every convergent series absolute convergent ? Justify your answer. Test for convergence of the series

$$1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} - \dots$$

~~(7,11.75)~~

3. (a) Test for the convergence of the series

i) $\sum_{n=1}^{\infty} \left(1/n\right)^{1/n}$

ii) $\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \dots..$

(b) Obtain Maclaurin's series expansion of $f(x) = (1+x)^m \quad \forall \, x \in R$, where m is a positive integer.

~~(5+7, 6.75)~~

4. (a) State and prove Rolle's Theorem. Give its ~~Geometrical~~ geometrical ~~Interpretation~~interpretation. Further prove that if **p** is a polynomial and **p**$'$ the derivative of **p,** then between any two consecutive zeroes of **p**$'$, there lies at most one zero of **p.**

(b) Let f be the function defined on R by setting

$$f(x) = \begin{cases} |x-1| + |x+2|, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Examine the function f for continuity and derivability at x = 1 and x = -2.

~~(10.75,8)~~

5. (a) Derive a formula which can be used for interpolating a value of f(x) near the end of the tabulated values.

(b) Identify the following expression and derive ~~the same~~it by defining the appropriate conditions.

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + \cdots + (x - x_0)(x - x_1) \dots (x - x_n)f(x_0, x_1, \dots \dots, x_n)$$

(c) Use method of separation of symbols to prove the identities

$$u_0 + \binom{n}{1}u_1 x + \binom{n}{2}u_2 x^2 + \cdots + \binom{n}{n}u_n x^n = (1+x)^n u_0 + \binom{n}{1}(1+x)^{n-1} x \Delta u_0 + \binom{n}{2}(1+x)^{n-2} x^2 \Delta^2 u_0 + \cdots \dots \dots + x^n \Delta^n u_0$$

~~(7,7,4.75)~~

6 (a) Define~~d~~ the operators E, Δ, μ, δ and ▽ and show that

$$\mu = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) = \frac{2 + \Delta}{2\sqrt{1 + \Delta}} = \sqrt{1 + \frac{1}{4}\delta^2}$$

(b) Calculate approximations to the value of $\int_0^6 \frac{dx}{1+x^2}$ by using ~~simpson's~~ Simpson's one-third rule.

~~(9.75, 9)~~