Unique paper Code	:	32371303_OC
Name of the Paper	:	Mathematical Analysis
Name of the Course	:	B.Sc. (H) Statistics Under CBCS
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	75

Instructions for Candidates

1. **Attempt any four questions.**
2. **All Questions ~~carries~~ carry equal marks ~~18.75 marks each~~.**

1. (a) Define a lower bound and an infimum of a non-empty bounded set S of real numbers. Prove that a real number t is the infimum of S iff

(i) $x \geq t \quad \forall\ x \in S$

(ii) for each $\epsilon > 0$, there is a real number $x \in S$ such that $x < t + \epsilon$.

(b) Define limit point of a set. Prove that every infinite bounded set of real numbers has a limit point.

2. (a) Show that the set of rational numbers is not ordered complete.

(b) Define neighbourhood of a point. If M and N are neighbourhoods of a point p, then prove that M∩N is also a neighbourhood of p.

3. (a) Test for the convergence of the series

i) $\sum_{n=1}^{\infty} \left(1/n\right)^{1/n}$

ii) $\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \ldots..$

(b) Obtain Maclaurin's series expansion of $f(x) = (1+x)^m \quad \forall\ x \in R$~~,~~.

4. (a) State and prove Rolle's Theorem. Give its Geometrical Interpretation. Further prove that if **p** is a polynomial and **p**$'$ the derivative of **p,** then between any two consecutive zeroes of **p**$'$, there lies at most one zero of **p.**

(b) Let f be the function defined on R by setting

$$f(x) = \begin{cases} |x-1| + |x+2|, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Examine the function f for continuity and derivability at x = 1 and x = -2.

5. (a) Derive a formula which can be used for interpolating a value of f(x) near the end of the tabulated values.

(b) Use method of separation of symbols to prove ~~the identities~~that

$$u_0 + \binom{n}{1}u_1 x + \binom{n}{2}u_2 x^2 + \cdots + \binom{n}{n}u_n x^n = (1+x)^n u_0 + \binom{n}{1}(1+x)^{n-1} x \Delta u_0 + \binom{n}{2}(1+x)^{n-2} x^2 \Delta^2 u_0 + \cdots \ldots \ldots + x^n \Delta^n u_0$$

6 (a) Defined the operators E, Δ, μ, δ and ▿ and show that

$$\mu = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) = \frac{2+\Delta}{2\sqrt{1+\Delta}} = \sqrt{1 + \frac{1}{4}\delta^2}.$$

(b) Calculate approximations to the value of $\int_0^6 \frac{dx}{1+x^2}$ by using simpson's three eighth rule.