Question Bank

B.Sc.(H) Mathematics- II sem
Real Analysis

Q1. Let $A = \left\{n + \frac{(-1)^n}{n} \mid n \in N\right\}$. Compute *sup A* and *inf A* if they exist. Also give proof.

Q2. By using the definition of limit, find the limit:

$$\lim_{n\to\infty} \sqrt{n^2 + \sin(n!)} - n$$

Q3. Let a_n be an arithmetic progression where $a_n > 0 \ \forall\, n \in N$. Define

$$x_n = \frac{a_1}{1} + \frac{a_2}{2^2} + \frac{a_3}{3^2} + \cdots + \frac{a_n}{n^2}$$

By using Cauchy criterion, show that the sequence $\{x_n\}$ diverges.

Q4. Discuss the convergence of the sequence $\{x_n\}$ where

$$x_n = 1 + \frac{1}{2^k} + \frac{1}{3^k} + \cdots + \frac{1}{n^k},\ k \in N.$$

Q5. Check whether the following sequence $\{x_n\}$ is convergent

$$x_1 = \frac{3}{2}\,,\ x_{n+1} = 2 - \frac{1}{x_n},\ \text{for } n \geq 1.$$

If yes, find the limit.

Q6. Suppose $\{x_n\}$ is a sequence of positive real numbers and $y_n = \frac{x_n}{1+x_n}$. Then which of the following are true? Explain each.

1. $\{x_n\}$ is convergent if $\{y_n\}$ is convergent.
2. $\{y_n\}$ is convergent if $\{x_n\}$ is convergent.
3. $\{x_n\}$ is bounded if $\{y_n\}$ is bounded.
4. $\{y_n\}$ is bounded if $\{x_n\}$ is bounded.

Also, if $\sum x_n = L$ where L is finite and $y_n = x_n + x_{n+1} + x_{n+3}$, then does the series $\sum y_n$ converges? If yes, find the sum.

Q7. Check the convergence or divergence of the following sequence:

(i) $x_n = \frac{cos\ 1}{3} + \frac{cos\ 3}{3^2} + \cdots + \frac{\cos 2n-1}{3^n}$, $n \in N$.

(ii) $x_{n+1} = 1 - \sqrt{1 - x_n}\ \ \forall\, n \geq 1, x_1 < 1$

Q8. Check whether the following sequence $\{x_n\}$ is convergent

$$x_1 = \frac{3}{2}\,,\ x_{n+1} = 2 - \frac{1}{x_n},\ \text{for } n \geq 1.$$

If yes, find the limit.

Q9. Check the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}+\log n}{n^{3/2}}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n^{\ln n}}$