Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351303_OC**
Name of Paper	: **C 7-Multivariate Calculus**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Let $z(x, y)$ be a function of two independent variables x and y. How many second order partial derivatives of $z(x, y)$ may exist? Write expressions for all these partial derivatives using the increment h in x and k in y. Let

$$f(x, y, z) = x^3yz + y^3zx + z^3xy,$$

x, y and z being independent variables, find functions $A(x, y, z)$, $B(x, y, z)$, $C(x, y, z)$ and the constant m such that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = x^2 A(x, y, z) + y^2 B(x, y, z) + z^2 C(x, y, z)$$

and

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = mxyz.$$

Further, assuming that the limit exists, show that

$$\lim_{(x,y,z)\to(0,0,0)} \frac{f(x, y, z)}{(x^2 + y^2 + z^2)^2} = 0.$$

2. Find the absolute extrema of the function $f(x, y) = 2\sin x + \cos y$ on S where S is the rectangular region with vertices $(2, 0), (2, 5), (-2, 0), (-2, 5)$.

How the result is affected and why if the rectangular region S is changed to be with vertices $(0, 0), (2, 0), (2, 5), (0, 5)$?

3. Verify Green's Theorem for the line integral

$$\oint_C (x^2 - 2xy)\, dx + (x^2 y + 3)\, dy$$

where the contour *C* is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

Also find the area enclosed by this closed curve *C*.

4. Using a suitable transformation evaluate the following integrals

(i) $\int_1^2 \int_{\frac{1}{y}}^{y} \sqrt{\frac{y}{x}}\, e^{\sqrt{xy}}\, dx\, dy$

(ii) $\int_0^1 \int_0^{1-x} \sqrt{(x+y)}\, (y-2x)^2\, dy\, dx.$

5. If $\vec{F}(x,y,z) = 2xy\,\hat{\imath} + yz^2\hat{\jmath} + xz\,\hat{k}$ and *S* is the surface of the region bounded by $x = 0,\; y = 0,$ $z = 0,\; y = 3$ and $x + 2z = 6$ then apply Divergence Theorem to evaluate

$$\iint_S \vec{F} \cdot \hat{N}\, dS.$$

Also find the volume of the region bounded by the surface *S*.

6. Find the work done in moving a particle in the force field

$$\vec{F}(x,y,z) = 3x^2\,\hat{\imath} + (2xz - y)\,\hat{\jmath} + z\,\hat{k}$$

along

(i) straight line from $A\ (0,0,0)$ to $B\ (2,1,3)$,

(ii) parametric curve *C*: $x = 2t^2, y = t, z = 4t^2 - t$ from $t = 0$ to $t = 1$,

(iii) cartesian curve *C*: $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.