## Question Bank

B.Sc(H) Mathematics-I Sem

Calculus

Q1. Find the slope of the line through the points $(-2,5)$ and $(7,1)$.
Q2. Find a point-slope equation of the line through the points $(1,3)$ and $(3,6)$.
Q3. Find the slope of the line having the equation $y-7=2(x-3)$ and find a point on the line
Q4. Find the slope-intercept equation of the line through the points $(2,4)$ and $(4,8)$.
Q5. Show that every line has an equation of the form $A x+B y=C$, where $A$ and $B$ are not both 0 , and that, conversely, every such equation is the equation of a line.

Q6. Find an equation of the line $L$ through $(-1,4)$ and parallel to the line $M$ with the equation $3 x+4 y$ $=2$.

Q7. Show that the lines parallel to a line $A x+B y=C$ are those lines having equations of the form $A x$ $+B y=E$ for some $E$. (Assume that $B \neq 0$.)

Q8. Find the slope-intercept equation of the line $M$ through $(1,4)$ that is perpendicular to the line $L$ with equation $2 x-6 y=5$.

Q9. Determine k so that the points $\mathrm{A}(7,5), \mathrm{B}(-1,2)$, and $\mathrm{C}(\mathrm{k}, 0)$ are the vertices of a right triangle with right angle at $B$.

Q10. Show that, if a graph is symmetric with respect to both the $x$-axis and the $y$-axis, then it is symmetric with respect to the origin.

Q11. Show that the product of two even functions and the product of two odd functions are even functions.

Q12. Show that the product of an even function and an odd function is an odd function.
Q13. Find any vertical and horizontal asymptotes of the graph of the function $f(x)=(4 x-5) /(3 x+2)$.
Q14. Determine the points of discontinuity (if any) of the function $f(x)$ such that $f(x)=x^{2}$ if $x \neq 0$ and $\mathrm{f}(\mathrm{x})=\mathrm{x}$ if $\mathrm{x}>0$.

Q15. Determine the points of discontinuity (if any) of the function $f$ such that $f(x)=1$ if $x$ is rational and $f(x)=0$ if $x$ is irrational.

Q16. Find the slope-intercept equation of the tangent line to the graph of the function $f(x)=4 x 3-7 x 2$ at the point corresponding to $\mathrm{x}=3$.

Q17. Specify all lines through the point $(1,5)$ and tangent to the curve $y=3>x 3+x+4$.

Q18. If the line $4 x-9 y=0$ is tangent in the first quadrant to the graph of $y=(1 / 2) x^{3}+c$, what is the value of $c$ ?

Q19. Where does the normal line to the curve $y=x-x^{\wedge} 2$ at the point $(1,0)$ intersect the curve a second time?

Q20. Find the angle of intersection of the lines L1: $y=x-3$ and L2: $y=-5 x+4$.

Q21. Find the angle of intersection of the tangent lines to the curves $x y=1$ and $y=x^{\wedge} 3$ at the common point $(1,1)$.

Q22. Prove that, if $f^{\prime}(x)>0$ for all $x$ in the open interval $(a, b)$, then $f(x)$ is an increasing function on ( $a$, b).

Q23. Let $f(x)$ be a differentiable function such that $f^{\prime}(x)^{\wedge} 0$ for all $x$ in the open interval $(a, b)$. Prove that there is at most one zero of $f(x)$ in $(a, b)$.

Q24. Consider the polynomial $f(x)=5 x 3-2 x 2+3 x-4$. Prove that $f(x)$ has a zero between 0 and 1 that is the only zero of $f(x)$.

Q25. Let $F(x)$ and $g(x)$ be differentiable functions such that $f(a) \geq g(a)$ and $f^{\prime}(x)>g^{\prime}(x)$ for all $x$. Show that $f(x)>g(x)$ for all $x>a$.

Q26. Use the mean value theorem to prove that $\tan \mathrm{x}>\mathrm{x}$ for $0<\mathrm{x}<\pi / 2$.

Q27. Find a point on the graph of $y=x^{\wedge} 2+x+3$, between $x=1$ and $x=2$, where the tangent line is parallel to the line connecting $(1,5)$ and $(2,9)$.

Q28. Prove that the zeros of $\sin x$ and $\cos x$ separate each other; that is, between any two zeros of sin $x$, there is a zero of $\cos x$, and vice versa.

Q29. At the point $(1,2)$ of the curve $x^{\wedge} 2-x y+y^{\wedge} 2=3$, find an equation of the tangent line.
Q30. For the curve $y^{\wedge} 3=x^{\wedge} 2$, calculate $y^{\prime}(a)$ by implicit differentiation and (b) by first solving for $y$ and then differentiating. Show that the two results agree.

Q31. At the point $(1,2)$ of the curve $x^{\wedge} 2-x y+y^{\wedge} 2=3$, find the rate of change with respect to $x$ of the slope of the tangent line to the curve.

Q32. Plot the function $f(x)=\operatorname{Sin}[x]$. Highlight the discontinuities in the function. Also verify it by showing through existence of Limit at any point of discontinuity.

Q33. Find the characteristic equation and its roots. Also verify Cayley-Hamilton Theorem and find the Inverse of the matrix;

$$
\begin{array}{ccc}
2 & -1 & 1 \\
-1 & 2 & -1 \\
1 & -1 & 2
\end{array}
$$

Q34. Solve the system of equations using Gauss elimination. Also comment on the type of solution on the basis of rank. Verify your solution using Solve Function.
$x-3 y-7 z=4$
$-2 x-5 y=3$
$4 x-1 y-2 z=5$.
Q35. Sketch the Polar curve $r=1-2 \cos \theta$. Also determine the area covered by them.
Q36. Obtain a Solid by revolving the region bounded by the curves $\mathrm{y}=\mathrm{x}^{2}$ and $\mathrm{y}=\mathrm{x}$ about x -axis. Determine the Volume of the solid.

